

Q1

1a

Reduce the power by one and multiply by the original power

$$\frac{dy}{dx} = 4x^3 \quad []$$

1b

Reduce the power by one and multiply by the original power

$$\frac{dy}{dx} = (-3) \times 2x^{-4}$$

$$\frac{dy}{dx} = -6x^{-4} \text{ or } -\frac{6}{x^4} \quad []$$

The question does not ask for the question in a particular form so either of the above are fine

1c

Rewrite as powers using the laws of indices

$$y = 4x^{-1}$$

And differentiate

$$\frac{dy}{dx} = (-1) \times 4x^{-2}$$

$$\frac{dy}{dx} = -4x^{-2} \text{ or } -\frac{4}{x^2} \quad []$$

Q2

For each term, reduce the power by 1 and multiply by the original power

$$\frac{dy}{dx} = 3 \times 4x^2 + 2x^0$$

$$\frac{dy}{dx} = 12x^2 + 2 \quad []$$

2b

Reduce the power by one and multiply by the original power

$$\frac{dy}{dx} = (-2) \times -5x^{-3}$$

$$\frac{dy}{dx} = -10x^{-3} \text{ or } -\frac{10}{x^3} \quad []$$

Either version of the answer is accepted

2c

Rewrite as powers using the laws of indices

$$y = \frac{1}{3} \times \frac{1}{x} = \frac{1}{3}x^{-1}$$

And differentiate

$$\frac{dy}{dx} = (-1) \times \frac{1}{3}x^{-2}$$

$$\frac{dy}{dx} = -\frac{1}{3}x^{-2} \text{ or } -\frac{1}{3x^2} \quad []$$

Q3

For each term, reduce the power by 1 and multiply by the original power. The constant term, 4, disappears

$$\frac{dy}{dx} = 3 \times 2x^2 - 2 \times 6x^1 + 3x^0$$

Simplify

$$\frac{dy}{dx} = 6x^2 - 12x + 3 \quad []$$

3b

Rewrite as powers using the laws of indices

$$-\frac{5}{3x^4} = -\frac{5}{3} \times \frac{1}{x^4} = -\frac{5}{3}x^{-4}$$

[]

Reduce the power by 1 and multiply the original power

$$(-4) \times \left(-\frac{5}{3}x^{-5}\right)$$

$$\frac{20}{3}x^{-5} \text{ or } \frac{20}{3x^5} \quad []$$

3c

Rewrite the last term as powers using the laws of indices

$$\frac{2}{3}x^2 + \frac{1}{5}x - \frac{3}{2}x^{-1}$$

[]

Rewrite the last term as powers using the laws of indices

$$\frac{2}{3}x^2 + \frac{1}{5}x - \frac{3}{2}x^{-1}$$

[]

Differentiate, taking care with fractions

$$\frac{4}{3}x + \frac{1}{5} + \frac{3}{2}x^{-2} \text{ or } \frac{4}{3}x + \frac{1}{5} + \frac{3}{2x^2} \quad []$$

Q4

For each term, reduce the power by 1 and multiply by the original power. The constant term, 11, disappears

$$\frac{dy}{dx} = 2 \times 2x^1 - 6x^0$$

Simplify

$$\frac{dy}{dx} = 4x - 6$$

one term correct []
both terms correct []

4b

Equate the derivative (which is the gradient function) found in part (a) to 6

$$4x - 6 = 2$$

[]

Solve for x

$$4x = 8$$

$$x = 2$$

The question asks for the coordinates, so the y coordinate is also needed. Substitute the found x value into the **original equation** for y

$$y = 2(2)^2 - 6(2) - 11$$

$$= -15$$

Write as a coordinate point

$$(2, -15) \quad []$$

Q5

For each term, reduce the power by 1 and multiply by the original power. The constant term, 9, disappears

$$\frac{dy}{dx} = 3x^2 + 2 \times \frac{7}{2}x^1 - 2x^0$$

Simplify

$$\frac{dy}{dx} = 3x^2 + 7x - 2$$

one term correct [1]

fully correct answer [1]

5b

- i) Substitute $x = -3$ into the derivative (which is the gradient function) found in part (a)

$$\frac{dy}{dx} = 3(-3)^2 + 7(-2) - 2$$

[1]

And evaluate, taking care with negative numbers—put them in brackets if using a calculator

$$= 27 - 14 - 2$$

$$= 4$$

gradient = 4 [1]

- ii) Substitute $x = -2/3$ into the derivative (which is the gradient function) found in part (a)

$$\frac{dy}{dx} = 3\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) - 2$$

[1]

And evaluate, taking care with fractions—put them in brackets if using a calculator

$$= \frac{4}{3} - \frac{14}{3} - 2$$

$$= 4$$

gradient = 4 [1]

5c

The gradients are equal therefore

the tangents are parallel [1]

Q6

In kinematics, velocity is the rate of change of displacement
Differentiate the displacement function with respect to x

$$v = \frac{ds}{dt} = 18t^2 - 24t$$

one term correct [1]

fully correct expression for v [1]

In kinematics, acceleration is the rate of change of velocity
Differentiate the velocity function with respect to x

$$a = \frac{dv}{dt} = 36t - 24$$

one term correct [1]

fully correct expression for a [1]

6b

Equate the expression for acceleration found in part (a) to 3

$$36t - 24 = 3$$

[1]

Solve

$$36t = 27$$

$$t = \frac{27}{36}$$

$t = 0.75$ seconds [1]

Q7

Differentiate each term.

$$\frac{dy}{dx} = 3 \times 5x^2 - 2x^1 - 6x^0$$

$$\frac{dy}{dx} = 15x^2 - 2x - 6$$

two terms correct (out of three) and no extra terms such as "4" []
fully correct answer []

7b

The gradient is given by $\frac{dy}{dx}$ for a given value of x . So equate $\frac{dy}{dx}$ to 2.

$$15x^2 - 2x - 6 = 2$$

[]

Start by subtracting 2 from both sides to leave a quadratic equal to zero.

$$15x^2 - 2x - 8 = 0$$

[]

Solve using your preferred method; quadratic formula, completing the square or factorisation (the question doesn't ask for any rounding, which is a clue that the quadratic will factorise). Here we will use the quadratic formula as it looks like a trickier factorisation.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 15 \times (-8)}}{2 \times 15}$$

correct first step to solving using any of the above methods []

$$x = \frac{2 \pm \sqrt{484}}{30}$$

Solve using your preferred method; quadratic formula, completing the square or factorisation (the question doesn't ask for any rounding, which is a clue that the quadratic will factorise). Here we will use the quadratic formula as it looks like a trickier factorisation.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 15 \times (-8)}}{2 \times 15}$$

correct first step to solving using any of the above methods []

$$x = \frac{2 \pm \sqrt{484}}{30}$$

$$x = \frac{4}{5}, \quad x = -\frac{2}{3} \quad []$$

Both answers must be stated (as fractions or decimals) for the final mark

Q8

Differentiate each term.

$$\frac{dy}{dx} = 3 \times x^2 - 2 \times 6x^1 - 15x^0$$

$$\frac{dy}{dx} = 3x^2 - 12x - 15$$

two terms correct [1]
fully correct answer [1]

8b

Stationary points occur where $\frac{dy}{dx} = 0$ so equate $\frac{dy}{dx}$ to 0.

$$3x^2 - 12x - 15 = 0$$

[1]

Divide both sides by the common factor 3 to leave a quadratic equation in the form $x^2 + bx + c = 0$

$$x^2 - 4x - 5 = 0$$

This is a straightforward quadratic to factorise, however you could solve using the quadratic formula or by completing the square if you wish.

$$(x - 5)(x + 1) = 0$$

correct first step to solving, using any of the above methods [1]

$$x = 5, x = -1$$

both x values [1]

Be careful! You are asked for the coordinates, not just the x value- so substitute each value of x into the equation of the curve to find the corresponding y value.

$$y = (-1)^3 - 6(-1)^2 - 15(-1) = 8$$

$$y = (5)^3 - 6(5)^2 - 15(5) = -100$$

Be careful! You are asked for the coordinates, not just the x value- so substitute each value of x into the equation of the curve to find the corresponding y value.

$$y = (-1)^3 - 6(-1)^2 - 15(-1) = 8$$

$$y = (5)^3 - 6(5)^2 - 15(5) = -100$$

(-1, 8) and (5, -100) [1]

Both coordinate points must be stated for the final mark

Q9

Differentiate each term by multiplying by the power and reducing the power by one.

$$\frac{dy}{dx} = 3 \times \frac{1}{3} x^2 - 1 \times 9x^0$$

$$\frac{dy}{dx} = x^2 - 9$$
 [2]

one mark for each correct term

9b

The gradient at any value of x is given by $\frac{dy}{dx} = x^2 - 9$. So the gradient is negative where $\frac{dy}{dx} < 0$.

$$x^2 - 9 < 0$$

[1]

To solve a quadratic inequality, first ignore the inequality sign and solve as an equation.

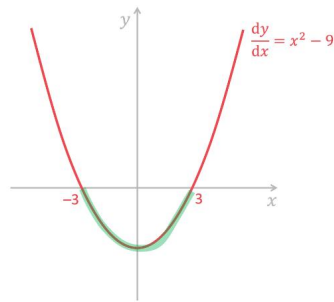
$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

Now you have the two roots, sketch the graph of $\frac{dy}{dx}$ and mark the region where $\frac{dy}{dx} < 0$.

Now you have the two roots, sketch the graph of $\frac{dy}{dx}$ and mark the region where $\frac{dy}{dx} < 0$.



$$-3 < x < 3 \quad [2]$$

1 mark for only " $x > -3$ " or " $x < 3$ "

Q10

The gradient of the curve at a point can be determined by finding the derivative of the equation of the curve, and substituting the x coordinate in.

Differentiate each term separately using the formula for differentiation.

$$y = ax^n \Rightarrow \frac{dy}{dx} = n \times ax^{n-1}$$

To differentiate $y = -x^2$, bring the 2 in front of x and reduce the power by 1.

$$\text{If the power is 1 then the derivative is just the coefficient of } x, y = 5x \Rightarrow \frac{dy}{dx} = 5$$

$$\text{If the power is 0 then the derivative is 0, } y = 24 \Rightarrow \frac{dy}{dx} = 0$$

Use this to find the derivative of each term.

$$\begin{aligned} y &= 24 + 5x - x^2 \\ \frac{dy}{dx} &= 0 + 5 + 2 \times (-x^{2-1}) \\ &= 5 - 2x \end{aligned}$$

1 mark for each correct term [2]

Find the gradient at the point where $x = -1.5$ by substituting it into the derivative.

$$5 - 2(-1.5) = 5 - (-3)$$

8 [1]

Q11

Formula for differentiation:

$$\begin{aligned} y &= ax^b \\ \frac{dy}{dx} &= abx^{b-1} \end{aligned}$$

To differentiate the first term, 6, remember that the derivative of any constant term is zero.

$$\begin{aligned} y &= 6 \\ \frac{dy}{dx} &= 0 \end{aligned}$$

Differentiate the second term, $4x$.

$$\begin{aligned} y &= 4x \\ \frac{dy}{dx} &= (4 \times 1)x^{1-1} = 4 \end{aligned}$$

Differentiate the third term, $-x^2$.

$$\begin{aligned} y &= -x^2 \\ \frac{dy}{dx} &= (-1 \times 2)x^{2-1} = -x^1 = -x \end{aligned}$$

$$\frac{dy}{dx} = 4 - 2x$$

Finding either term correctly [1]

$$y = -x^2$$

$$\frac{dy}{dx} = (-1 \times 2)x^{2-1} = -x^1 = -x$$

$$\frac{dy}{dx} = 4 - 2x$$

Finding either term correctly [1]
Both terms correct [1]

11b

To find the turning point, set $\frac{dy}{dx}$ equal to zero.

$$\frac{dy}{dx} = 0$$

$$4 - 2x = 0$$

[1]

Solve by subtracting 4 and dividing by -2.

$$-2x = -4$$

$$x = 2$$

Find the y coordinate by substituting the value of x into the equation for the curve.

$$y = 6 + 4(2) - (2)^2$$

$$= 6 + 8 - 4$$

$$= 10$$

(2, 10) [1]